Polynomial division:

Question: When might we want to divide one polynomial by another?

For example, dividing $f(x) = x^4 + 6x^3 + 11x^2 + 12x + 4$ by $g(x) = x^2 + 3x + 2$

Answer: One reason might be to find the roots (values of x that make f(x) = 0).

You could use a process called synthetic division to find the roots of a polynomial.

However, if you find synthetic division confusing and/or difficult to remember, then you can also use simple division.

Example: Suppose we want to find the roots of the polynomial

$$f(x) = 6x^4 + 17x^3 - 29x^2 - 2x + 8$$

The roots of this polynomial are given by $\pm\frac{P}{Q}$, where P is a factor of 8 and Q is a factor of 6

The factors of 8 are: 1 x 8, 2 x 4 The factors of 6 are: 1 x 6, 2 x 3

So the possible values of $\frac{P}{Q}$ are $\pm 8, \pm 4, \pm 2, \pm 1, \pm\frac{1}{2}, \pm\frac{8}{3}, \pm\frac{4}{3}, \pm\frac{2}{3}, \pm\frac{1}{3}, \pm\frac{1}{6}$

To see if x = 1 is a root, then we need to divide f(x) by (x – 1):

Start by writing the problem like one in long division: $x - 1 \overline{)\ 6x^4 + 17x^3 - 29x^2 - 2x + 8}$

Divide $6x^4$ by x to get $6x^3$. Write that above the horizontal line:

$$\begin{array}{r} 6x^3 \\ x - 1 \overline{)\ 6x^4 + 17x^3 - 29x^2 - 2x + 8} \end{array}$$

Now multiply $(x - 1)$ by $6x^3$, write below f(x), and subtract:

$$\begin{array}{r} 6x^3 \\ x - 1 \overline{)\ 6x^4 + 17x^3 - 29x^2 - 2x + 8} \\ 6x^4 - 6x^3 \downarrow \\ \hline 23x^3 - 29x^2 \end{array}$$

After subtracting, bring down the next term: $-29x^2$

Now divide $23x^3$ by x to get $23x^2$ and repeat:

$$\begin{array}{r} 6x^3 + 23x^2 \\ x - 1 \;\overline{)\; 6x^4 + 17x^3 - 29x^2 - 2x + 8} \\ \underline{6x^4 - 6x^3} \\ 23x^3 - 29x^2 \\ \underline{23x^3 - 23x^2} \\ -6x^2 - 2x \end{array}$$

Repeat again, dividing $-6x^2$ by x to get $-6x$:

$$\begin{array}{r} 6x^3 + 23x^2 - 6x \\ x - 1 \;\overline{)\; 6x^4 + 17x^3 - 29x^2 - 2x + 8} \\ \underline{6x^4 - 6x^3} \\ 23x^3 - 29x^2 \\ \underline{23x^3 - 23x^2} \\ -6x^2 - 2x \\ \underline{-6x^2 + 6x} \\ -8x + 8 \end{array}$$

And finally:

$$\begin{array}{r} 6x^3 + 23x^2 - 6x - 8 \\ x - 1 \;\overline{)\; 6x^4 + 17x^3 - 29x^2 - 2x + 8} \\ \underline{6x^4 - 6x^3} \\ 23x^3 - 29x^2 \\ \underline{23x^3 - 23x^2} \\ -6x^2 - 2x \\ \underline{-6x^2 + 6x} \\ -8x + 8 \\ \underline{-8x + 8} \\ 0 \end{array}$$

So the answer is:

$6x^4 + 17x^3 - 29x^2 - 2x + 8 = (x - 1)(6x^3 + 23x^2 - 6x - 8)$

Exercises:

Divide the following:

1. $(x^3 - 1)$ by $(x - 1)$

2. $(x^3 - 4x^2 - 20x + 48)$ by $(x - 2)$

3. $(x^3 + 6x^2 + 11x + 6)$ by $(x + 3)$

4. $(x^4 + 8x^3 + 24x^2 + 32x + 16)$ by $(x^2 + 4x + 4)$

Answers: 1. $x^2 + x + 1$ 2. $x^2 - 2x - 24$ 3. $x^2 + 3x + 2$ 4. $x^2 + 4x + 4$