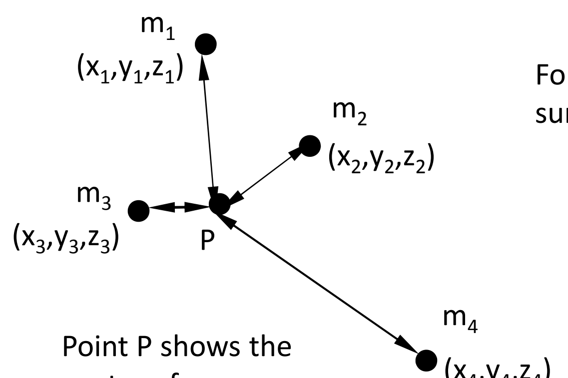
Center of Mass:

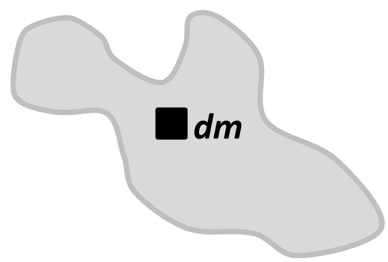
For a collection of N objects (in the figure N = 4), the position of the center of mass is given by

$$x_{com}(\text{or } y_{com} \text{ or } z_{com}) = \frac{\sum_{i=1}^{N} m_i x_i (\text{or } y_i \text{ or } z_i)}{m_{tot}} \qquad \text{where } m_{tot} = \sum_{i=1}^{N} m_i$$



Point P shows the center of mass

For a continuous distribution of mass, we replace the sums with integrals:



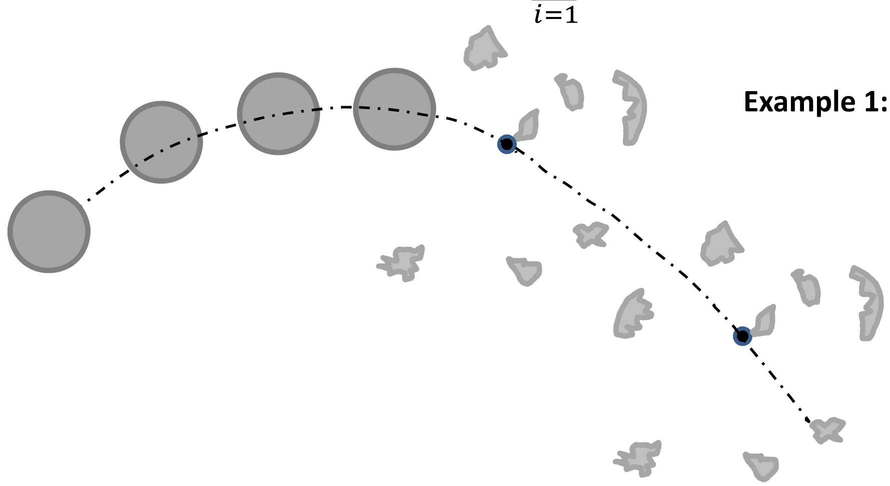
$$x_{com}(\text{or } y_{com} \text{ or } z_{com}) = \frac{\int dm\, x_i \;(\text{or } y_i \text{ or } z_i)}{m_{tot}} \qquad m_{tot} = \int dm$$

Question: Why is the "Center of Mass" concept important?

Answer: Because it greatly simplifies the motions of systems of objects (&/or objects with complicated shapes)

Why? Because the motion of the center of mass is determined by the net external force, i.e.

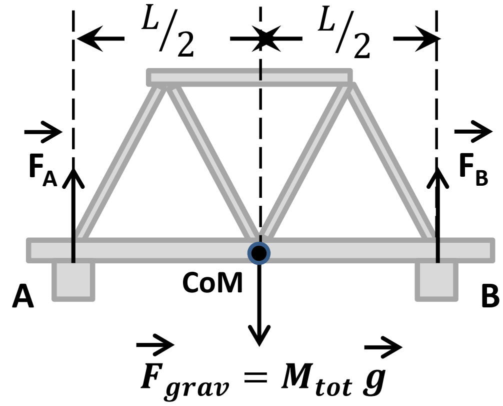
$$\sum_{i=1}^{N} m_i \vec{a}_i = m_{tot}\, \vec{a}_{com} = \vec{F}_{ext,net} = \sum_{i=1}^{N} \vec{F}_{ext,i}$$



Example 1: Exploding Shell

After the explosion, the center of mass continues to follow the original trajectory

Example 2: Bridge in Equilibrium



The sum of the forces and the sum of the torques must both equal zero.

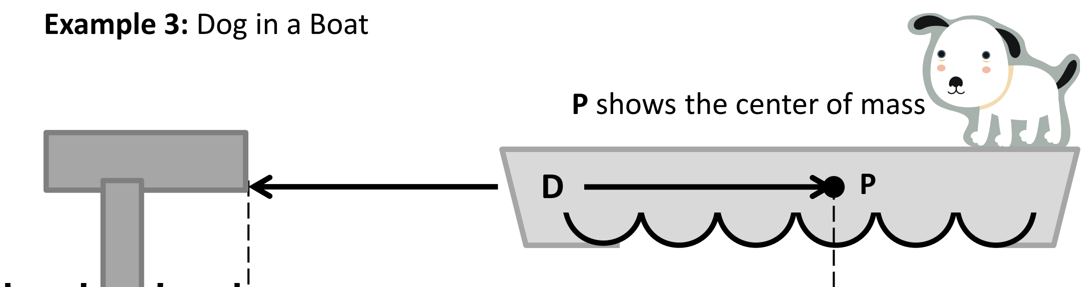
For calculating the total torque due to gravity, we can treat the bridge as a point mass located at the center of mass of the bridge.

So the equilibrium equations become: (taking the torques about point A)

$$F_A + F_B - M_{tot}g = 0$$

$$F_B L - M_{tot}\, g\left(\frac{L}{2}\right) = 0$$

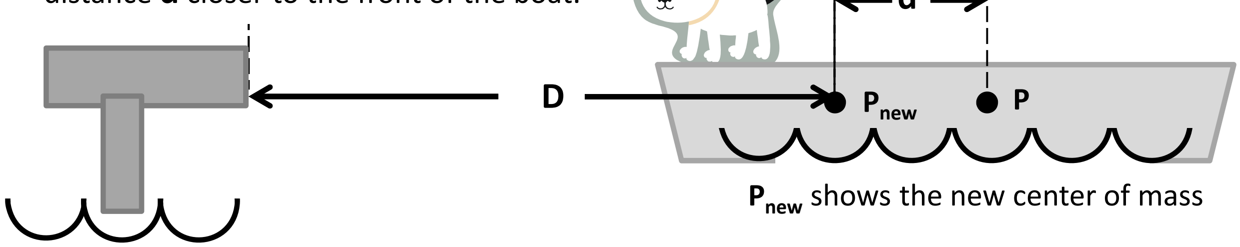
Example 3: Dog in a Boat



If the dog and boat are an isolated system, then their center of mass will not move with respect to external objects, even if the position of the dog changes.

For example, if the dog walks to the front of the boat, then the center of mass will also move closer to the front of the boat.

Here the center of mass has moved a distance **d** closer to the front of the boat.



P_{new} shows the new center of mass

However, the entire dog + boat system will move away from the dock by that same distance **d**, so the distance (**D**) between the center of mass and the front of the dock will not change.